

# 08中間演習解答

## 線形代数演習

### 問1 連立1次方程式を解け

$$(1) \begin{cases} x_1 + x_2 - x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 4 \\ -x_1 + 2x_2 - 4x_3 = -2 \end{cases}$$

$$\begin{aligned} & \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 2 & 1 & 3 & 4 \\ -1 & 2 & -4 & -2 \end{array} \right] \xrightarrow[\textcircled{3}+\textcircled{1}]{\textcircled{2}+\textcircled{1}\times(-2)} \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -1 & 5 & 2 \\ 0 & 3 & -5 & -1 \end{array} \right] \xrightarrow{\textcircled{2}\times(-1)} \left[ \begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & -5 & -2 \\ 0 & 3 & -5 & -1 \end{array} \right] \\ & \xrightarrow[\textcircled{3}+\textcircled{2}\times(-3)]{\textcircled{1}+\textcircled{2}\times(-1)} \left[ \begin{array}{ccc|c} 1 & 0 & 4 & 3 \\ 0 & 1 & -5 & -2 \\ 0 & 0 & 10 & 5 \end{array} \right] \xrightarrow{\textcircled{3}\times(\frac{1}{10})} \left[ \begin{array}{ccc|c} 1 & 0 & 4 & 3 \\ 0 & 1 & -5 & -2 \\ 0 & 0 & 1 & \frac{1}{2} \end{array} \right] \xrightarrow[\textcircled{2}+\textcircled{3}\times 5]{\textcircled{1}+\textcircled{3}\times(-4)} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \end{array} \right] \end{aligned}$$

(2)

$$\left[ \begin{array}{ccccc|c} 1 & -2 & 0 & 3 & 0 & 2 \\ 1 & -2 & 1 & 2 & 1 & 2 \\ 2 & -4 & 1 & 5 & 2 & 5 \end{array} \right] \xrightarrow[\textcircled{3}+\textcircled{1}\times(-2)]{\textcircled{2}+\textcircled{1}\times(-1)} \left[ \begin{array}{ccccc|c} 1 & -2 & 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 2 & 1 \end{array} \right]$$

$$\xrightarrow{\textcircled{3}+\textcircled{2}\times(-1)} \left[ \begin{array}{ccccc|c} 1 & -2 & 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array} \right] \xrightarrow{\textcircled{2}+\textcircled{3}\times(-1)} \left[ \begin{array}{ccccc|c} 1 & -2 & 0 & 3 & 0 & 2 \\ 0 & 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array} \right]$$

rank(A)=rank(A|b)=3  
自由度5-3=2

$x_2 = c_1, x_4 = c_2$ とおく

$x_5 = 1, x_3 = c_2 - 1, x_1 = 2c_1 - 3c_2 + 2$

$$x = \begin{bmatrix} 2c_1 - 3c_2 + 2 \\ c_1 \\ c_2 - 1 \\ c_2 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \quad c_1, c_2 \in R$$

## 問2 逆行列を求めよ

$a \neq 0$ に対して

$$\begin{aligned}
 & \left[ \begin{array}{ccc|ccc} 1 & 1 & -a+1 & 1 & 0 & 0 \\ 2 & 3 & 2a & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow[\textcircled{3}+\textcircled{1}\times(-1)]{\textcircled{2}+\textcircled{1}\times(-2)} \left[ \begin{array}{ccc|ccc} 1 & 1 & -a+1 & 1 & 0 & 0 \\ 0 & 1 & 4a-2 & -2 & 1 & 0 \\ 0 & 0 & a & -1 & 0 & 1 \end{array} \right] \\
 & \xrightarrow{\textcircled{1}+\textcircled{2}\times(-1)} \left[ \begin{array}{ccc|ccc} 1 & 0 & -5a+3 & 3 & -1 & 0 \\ 0 & 1 & 4a-2 & -2 & 1 & 0 \\ 0 & 0 & a & -1 & 0 & 1 \end{array} \right] \xrightarrow{\textcircled{3}\times\frac{1}{a}} \left[ \begin{array}{ccc|ccc} 1 & 0 & -5a+3 & 3 & -1 & 0 \\ 0 & 1 & 4a-2 & -2 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{a} & 0 & \frac{1}{a} \end{array} \right] \\
 & \xrightarrow[\textcircled{2}+\textcircled{3}\times(-4a+2)]{\textcircled{1}+\textcircled{3}\times(5a-3)} \left[ \begin{array}{ccc|ccc} 1 & 0 & -5a+3 & -2+\frac{3}{a} & -1 & 5-\frac{3}{a} \\ 0 & 1 & 0 & 2-\frac{2}{a} & 1 & -4+\frac{2}{a} \\ 0 & 0 & 1 & -\frac{1}{a} & 0 & \frac{1}{a} \end{array} \right]
 \end{aligned}$$

## 問3 行列式を求めよ.

簡単にしてから、計算してあげると良い.

$$\begin{aligned}
 (1) \quad & \begin{vmatrix} 5 & -3 & 14 \\ -5 & 6 & 7 \\ 10 & 3 & -7 \end{vmatrix} = 5 \cdot 3 \cdot 7 \begin{vmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 5 \cdot 3 \cdot 7 \begin{vmatrix} 1 & -1 & 2 \\ 0 & 1 & 3 \\ 0 & 3 & -5 \end{vmatrix} = 5 \cdot 3 \cdot 7(-5-9) = -1470 \\
 (2) \quad & \begin{vmatrix} 1 & -1 & 2 & 1 \\ 2 & -1 & 1 & 2 \\ -1 & 1 & 2 & 1 \\ 2 & 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 & 1 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 3 & -3 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 0 \\ 0 & 4 & 2 \\ 3 & -3 & -1 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 0 \\ 0 & 4 & 2 \\ 0 & 6 & -1 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 6 & -1 \end{vmatrix} = -16 \\
 (3) \quad & \begin{vmatrix} 3 & 2^2 & 1 & 1 \\ 3^2 & 2^3 & 1 & 7 \\ 3^3 & 2^4 & 1 & 7^2 \\ 3^4 & 2^5 & 1 & 7^3 \end{vmatrix} = \begin{vmatrix} 3 & 2^2 & 1 & 1 \\ 3^2-3^2 & 2^2(2-3) & 1-3 & 7-3 \\ 3^3-3^3 & 2^3(2-3) & 1-3 & 7(7-3) \\ 3^4-3^4 & 2^4(2-3) & 1-3 & 7^2(7-3) \end{vmatrix} = 3 \cdot 2^2 \cdot (-1) \cdot (-2) \cdot 4 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 7 \\ 2^2 & 1 & 7^2 \end{vmatrix} \\
 & = 96 \begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 5 \\ 0 & -1 & 7 \cdot 5 \end{vmatrix} = 96 \cdot (-35 + 5) = -2880
 \end{aligned}$$

### 問4 次元と1組の基底を求めよ

$$W = \left\{ x \in R^5 \left| \begin{bmatrix} 2 & 0 & -1 & 3 & 4 \\ 1 & 2 & 3 & 1 & -5 \\ 3 & 1 & 4 & -7 & 10 \end{bmatrix} x = 0 \right. \right\}$$

$$\begin{bmatrix} 2 & 0 & -1 & 3 & 4 \\ 1 & 2 & 3 & 1 & -5 \\ 3 & 1 & 4 & -7 & 10 \end{bmatrix} \xrightarrow{\textcircled{1} \leftrightarrow \textcircled{2}} \begin{bmatrix} 1 & 2 & 3 & 1 & -5 \\ 2 & 0 & -1 & 3 & 4 \\ 3 & 1 & 4 & -7 & 10 \end{bmatrix} \xrightarrow{\begin{matrix} \textcircled{2} + \textcircled{1} \times (-2) \\ \textcircled{3} + \textcircled{1} \times (-3) \end{matrix}} \begin{bmatrix} 1 & 2 & 3 & 1 & -5 \\ 0 & -4 & -7 & 1 & 14 \\ 0 & -5 & -5 & -10 & 25 \end{bmatrix}$$

$$\xrightarrow{\begin{matrix} \textcircled{2} \leftrightarrow \textcircled{3} \times (-\frac{1}{5}) \\ \textcircled{2} \leftrightarrow \textcircled{3} \times (-\frac{1}{5}) \end{matrix}} \begin{bmatrix} 1 & 2 & 3 & 1 & -5 \\ 0 & 1 & 1 & 2 & -5 \\ 0 & -4 & -7 & 1 & 14 \end{bmatrix} \xrightarrow{\begin{matrix} \textcircled{1} + \textcircled{2} \times (-2) \\ \textcircled{3} + \textcircled{2} \times 4 \end{matrix}} \begin{bmatrix} 1 & 0 & 1 & -3 & 5 \\ 0 & 1 & 1 & 2 & -5 \\ 0 & 0 & -3 & 9 & -6 \end{bmatrix}$$

$$\xrightarrow{\textcircled{3} \times (-\frac{1}{3})} \begin{bmatrix} 1 & 0 & 1 & -3 & 5 \\ 0 & 1 & 1 & 2 & -5 \\ 0 & 0 & 1 & -3 & 2 \end{bmatrix} \xrightarrow{\begin{matrix} \textcircled{1} + \textcircled{3} \times (-1) \\ \textcircled{2} + \textcircled{3} \times (-1) \end{matrix}} \begin{bmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 & -7 \\ 0 & 0 & 1 & -3 & 2 \end{bmatrix} \quad \begin{matrix} \text{rank}(A)=3 \\ \text{自由度} 5-3=2 \end{matrix}$$

$x_4 = c_1, x_5 = c_2$ とおく

$$x = \begin{bmatrix} -3c_2 \\ -5c_1 + 7c_2 \\ 3c_1 - 2c_2 \\ c_1 \\ c_2 \end{bmatrix} = c_1 \begin{bmatrix} 0 \\ -5 \\ 3 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -3 \\ 7 \\ -2 \\ 0 \\ 1 \end{bmatrix}$$

$\dim(W) = 2$ , ひとつの基底

$$\left\{ \begin{bmatrix} 0 \\ -5 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 7 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$$